# High-to-medium frequency mesoscale gravity waves in vertical winds and temperatures in the MLT & Comparison with high-resolution WACCM

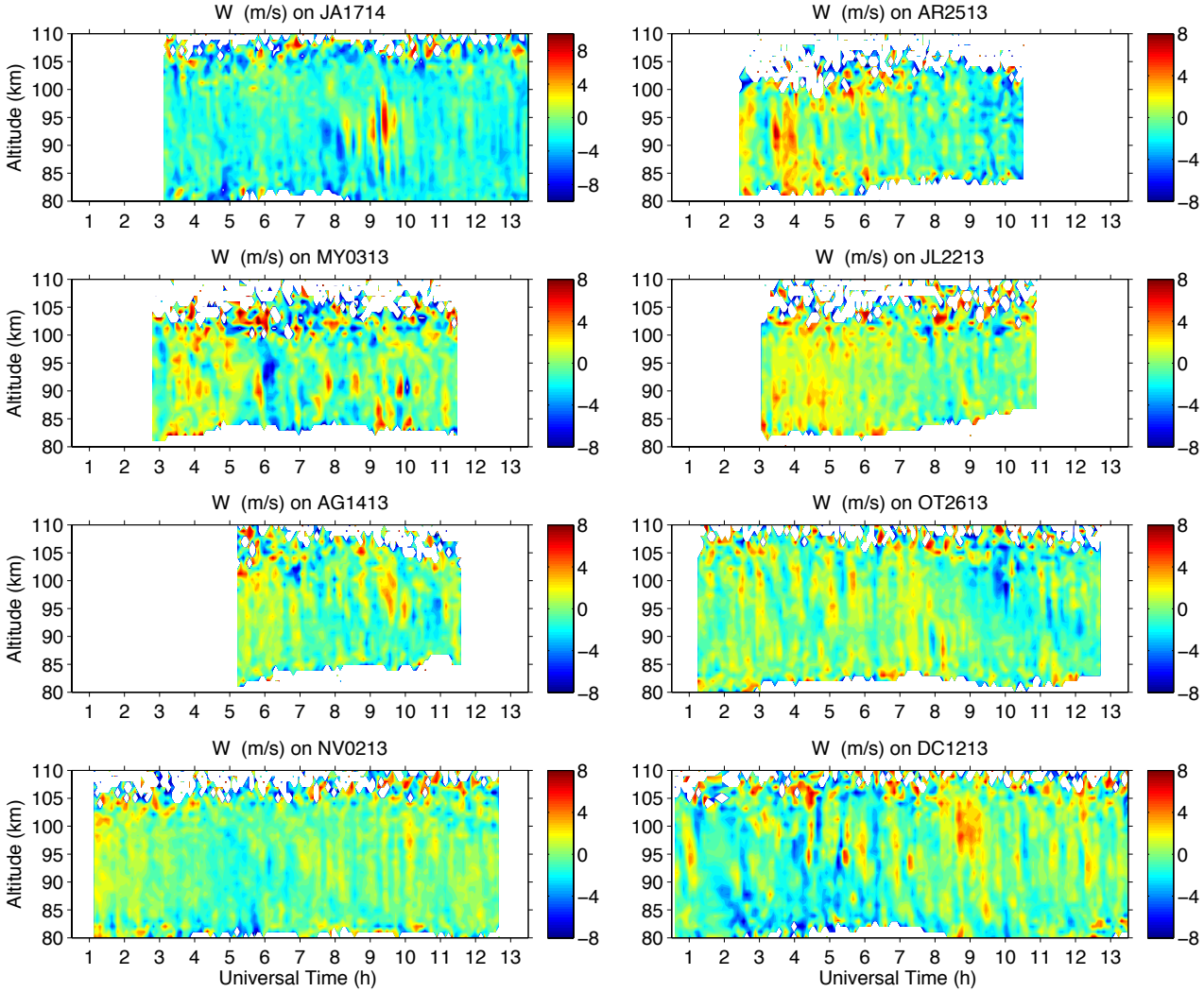
**Xian Lu[1], Xinzhao Chu[2], Haoyu Li[2], Cao Chen[2], John Smith[2], Sharon Vadas[3], Hanli Liu[4],**

*1. Clemson University, 2. University of Colorado Boulder, 3. NWRA, 4. HAO, NCAR*

ISWA, Tokyo, Japan, September 16

# Motivation

## 1. Persistent gravity waves with periods of 0.3-2.5 h in W




*Lu et al.*, [2016]

## 2. Mesoscale GWs, become resolvable in high-resolution GCMs.

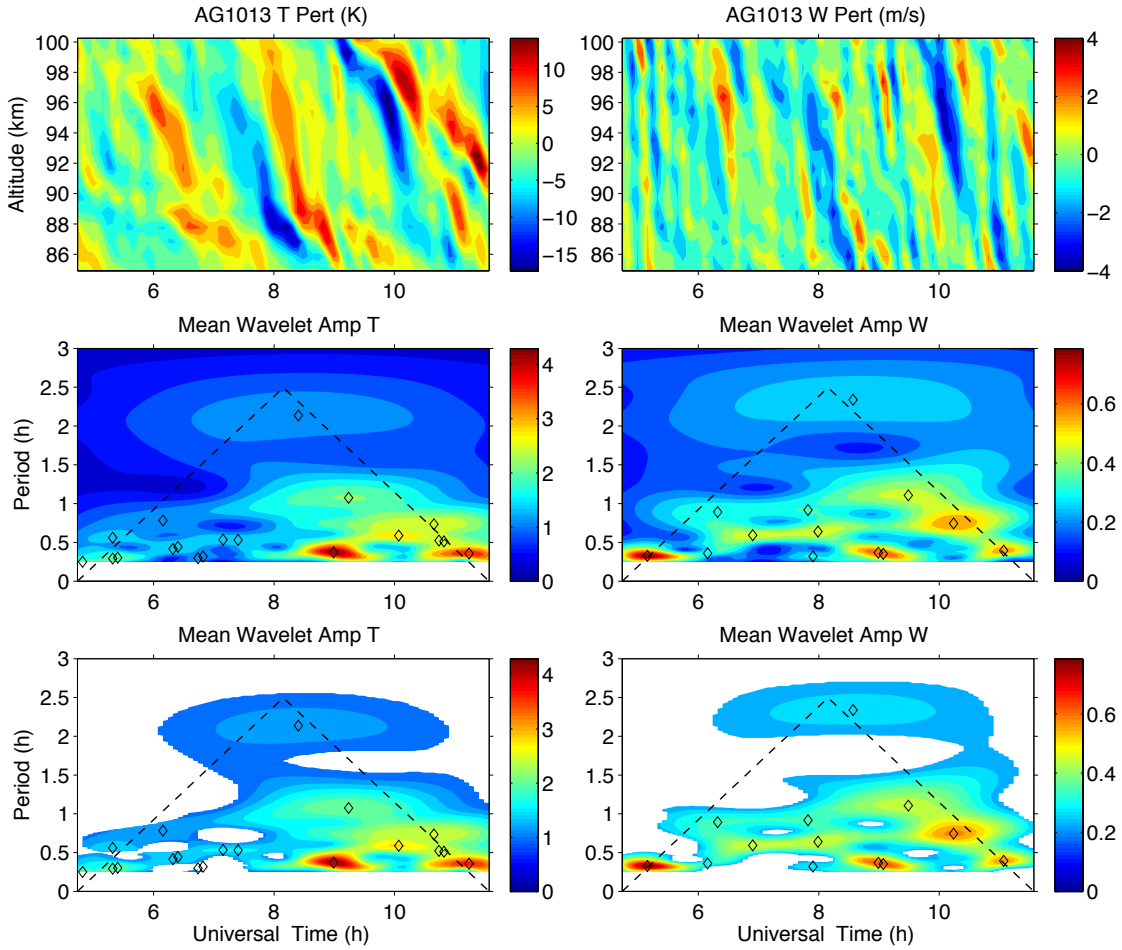
# Data Source

- Statistics of the STAR Na Doppler Lidar Data at Boulder, Colorado from April 2013 to January 2014.

| | April | May | June | July | August | September | October | November | December | January | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Night | 2 | 3 | 3 | 1 | 7 | 3 | 8 | 8 | 7 | 14 | 56 |
| Hour(h) | 15.9 | 18.5 | 12.9 | 6.9 | 42.2 | 23.0 | 73.1 | 73.6 | 69.2 | 125.5 | 460.8 |

- Data Resolution: $\Delta t$=7.5 min, $\Delta z$=0.96 km

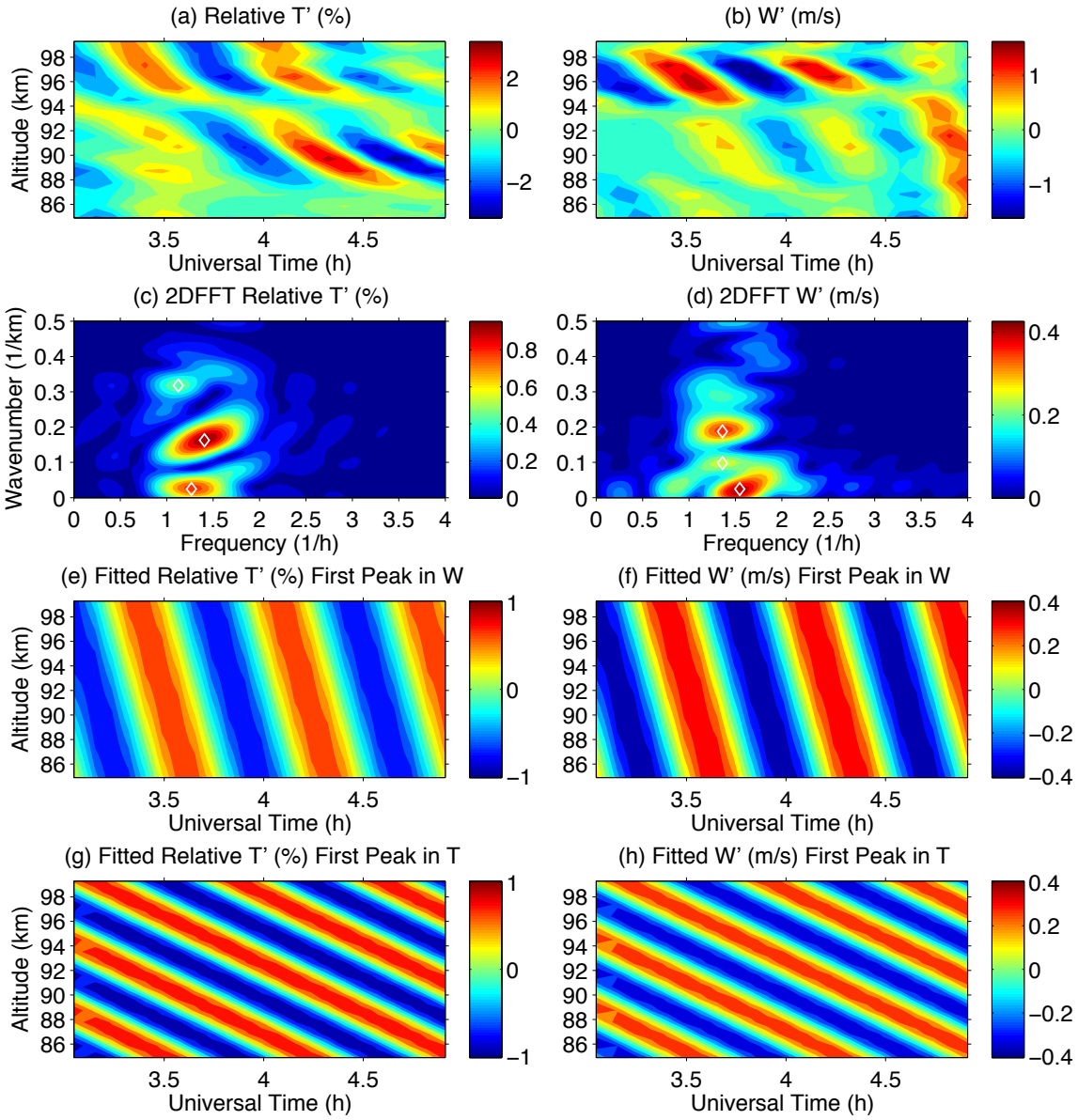
# Wavelet to Identify Waves



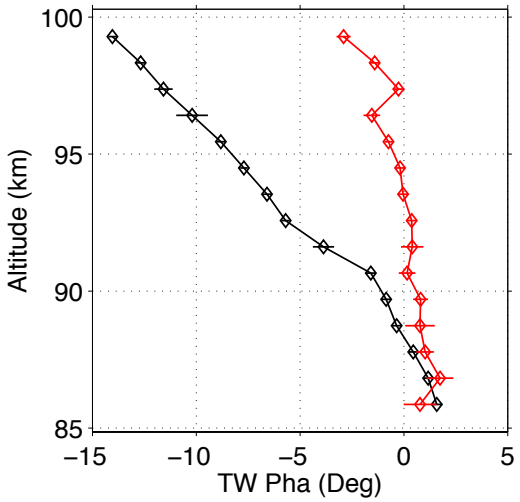

1. Identify local common peaks in T and W.
2. Extend local peaks for 3-4 wave cycles to derive amplitude and phase.
3. Require the peaks occur at 2/3 of the altitudes.

| | Wave 1 | Wave 2 | Wave 3 | Wave 4 | Wave 5 | Wave 6 |
|---|---|---|---|---|---|---|
| Period (h) | 2.3 | 1.1 | 0.78 | 0.31 | 0.37 | 0.38 |
| Duration (UT) | 5–12 | 7.7–11 | 9.1–11.5 | 4.7–5.6 | 8.4–9.6 | 10.4–11.6 |

# 2D FFT and 2D Fitting: Derivation of Wave Amplitude and Phase




**Phase: T, W**



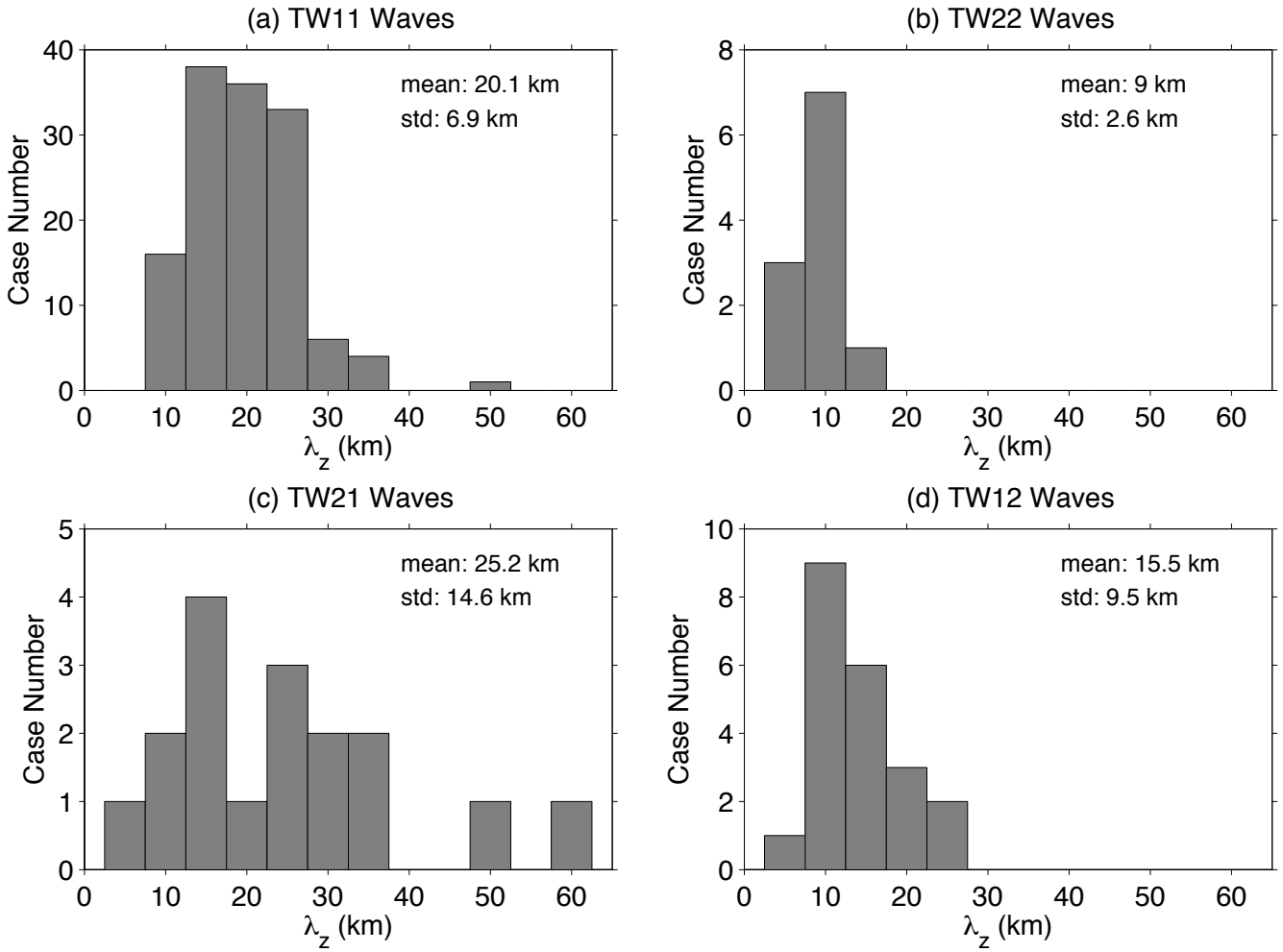

## 2D Fitting:

$$T'/\overline{T}(t,z) = A_T \cos(\overline{\omega}t - \overline{m}z - \varphi_T)$$

$$w'(t,z) = A_w \cos(\overline{\omega}t - \overline{m}z - \varphi_w)$$

*Lu et al.*, [2016]

# Vertical Wavelengths

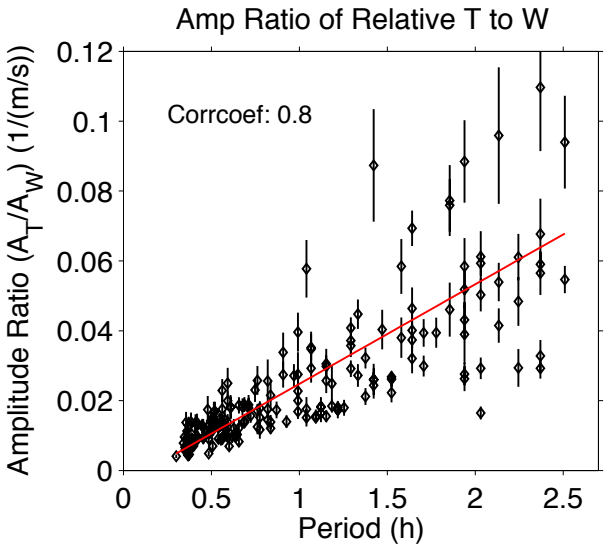


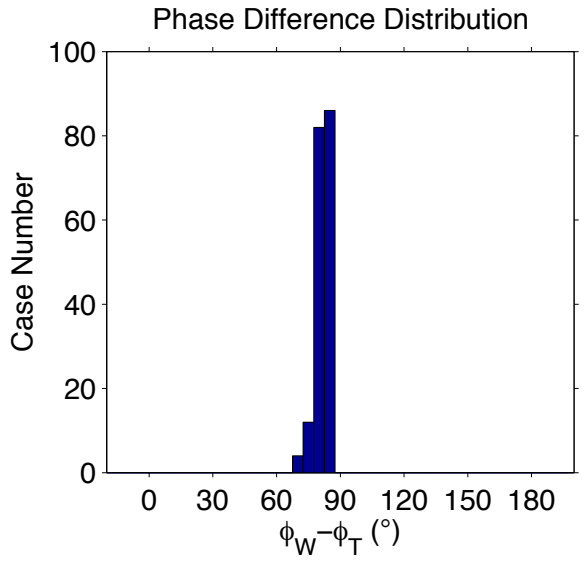

**TW11: 1st peak in $\lambda_z$(T)**
**1st peak in $\lambda_z$(W)**
**TW12: 1st peak in $\lambda_z$(T)**
**2nd peak in $\lambda_z$(W)**
**TW21: 2nd peak in $\lambda_z$(T),**
**1st peak in $\lambda_z$(W)**
**TW22: 2nd peak in $\lambda_z$(T),**
**2nd peak in $\lambda_z$(W)**

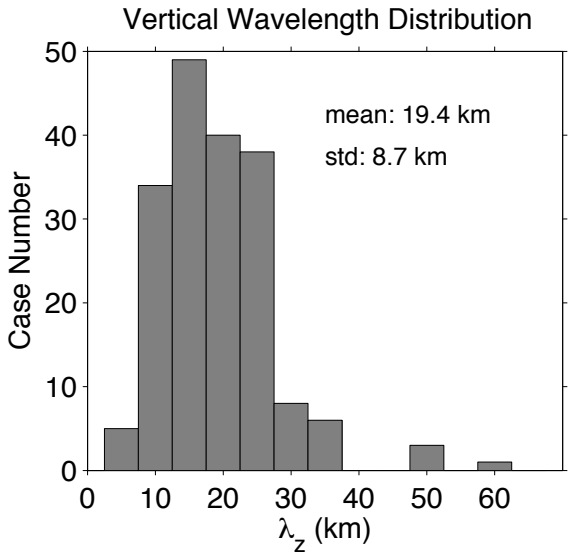
*Lu et al.*, [2016]

- The secondary waves usually have shorter vertical wavelengths than the primary waves.
- For the vertical wavelengths, the first peaks in vertical winds tend to be longer than those in temperatures.

# Amplitude Ratio & Phase Difference & Vertical Wavelength



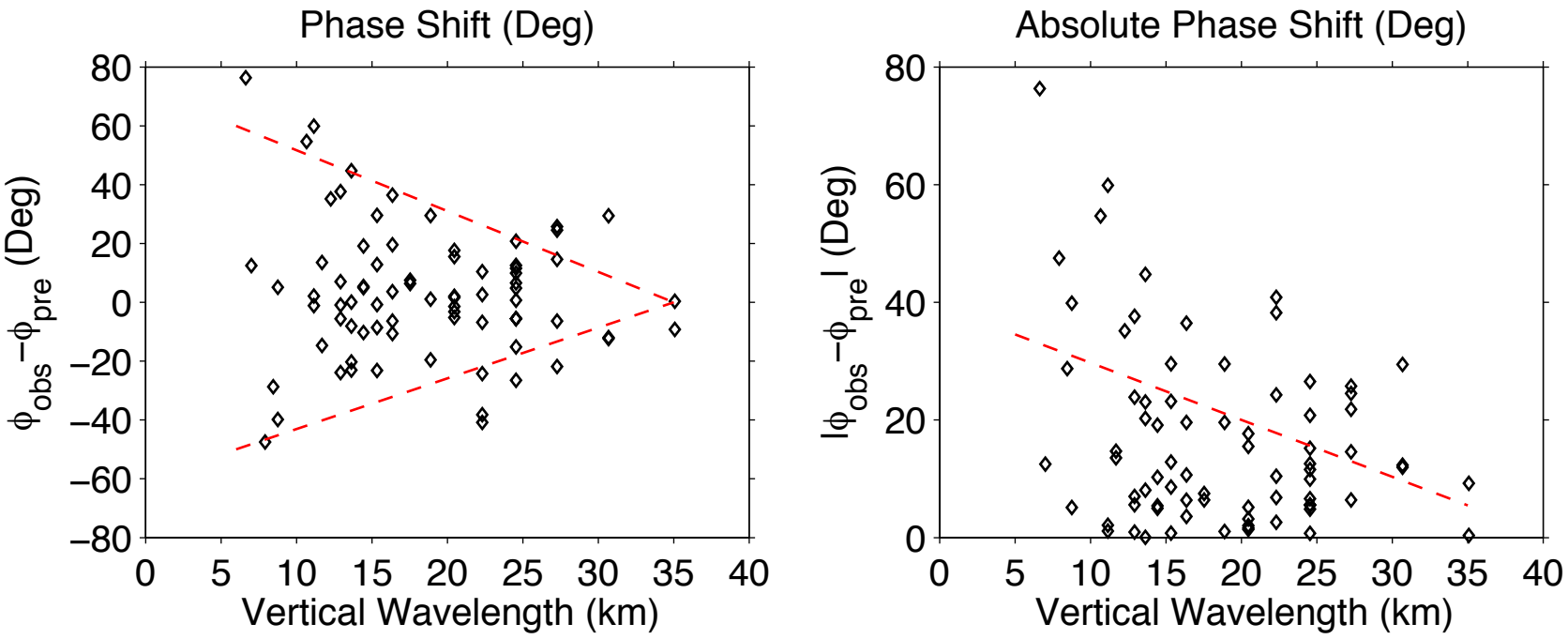

*Lu et al.*, [2016]

**Polarization relation without dissipation:**

$$\tilde{T}/\overline{T}/\tilde{w} = \frac{N^2\left(im-\frac{1}{2H}\right)-\frac{\hat{\omega}_I^2}{\gamma H}(1-\gamma)}{g\hat{\omega}_I\left(m-\frac{i}{2H}+\frac{i}{\gamma H}\right)} \approx \frac{N^2\left(im-\frac{1}{2H}\right)}{g\hat{\omega}_I\left(m-\frac{i}{2H}+\frac{i}{\gamma H}\right)} \approx \frac{iN^2}{g\hat{\omega}_I}$$

- Amplitude ratios have positive correlation with the period which is consistent with polarization relation of gravity waves.
- The phase differences tend to follow a Gaussian distribution (84.2°±26.7°).
- Vertical wavelengths are 19.4± 8.7 km.

# Phase Deviations from Non-Dissipation V.S. $\lambda_z$




*Lu et al.*, [2016]

- Compared with the phase differences for the waves without dissipations, the deviations can be both negative and positive.
- The absolute phase deviations appear to be statistically larger for shorter-$\lambda_z$ waves, and vice versa.

# Derive Intrinsic Period from Polarization Relation?

## Polarization relation with dissipation (Vadas and Nicolls [2012] ):

$$\tilde{T} / \overline{T} / \tilde{w} = \frac{(\gamma - 1)}{C_s^2 D}\left[\gamma i\omega_I (i\omega_I - \alpha K) + \frac{C_s^2}{H}(-im + \frac{1}{2H})\right]$$

$$D = \left[i\omega_I (\gamma im - \frac{1}{H} + \frac{\gamma}{2H}) + \frac{\gamma \alpha K}{\Pr}(-im + \frac{1}{2H})\right]$$

$$\alpha = -(k_H^2 + m^2) + \frac{1}{4H^2} - \frac{im}{H}$$

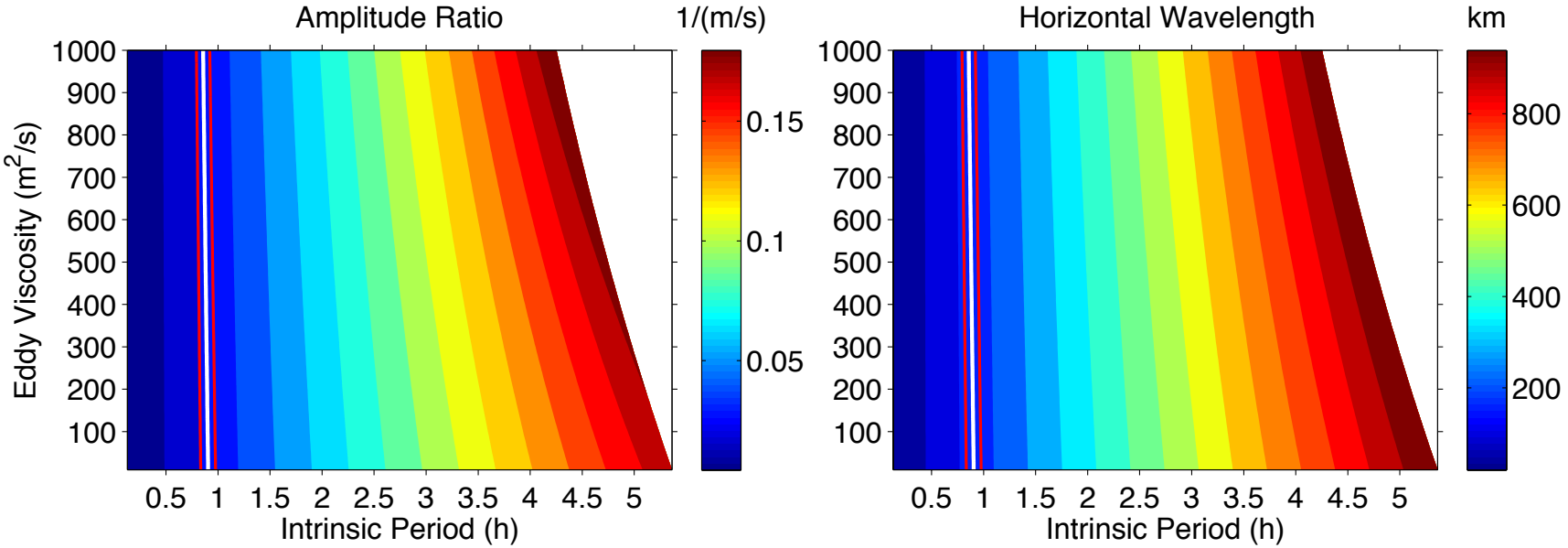
WKB approximation

Amplitude ratio is determined by:

**1: intrinsic frequency**

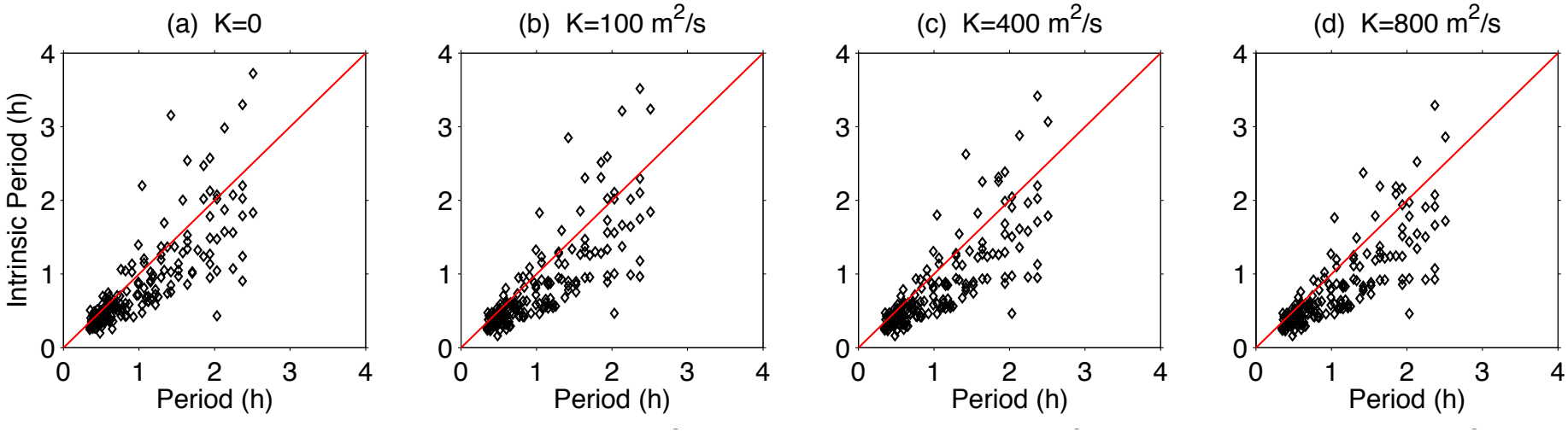
**2: eddy viscosity**

3: some known parameters




*Lu et al.*, [2016]

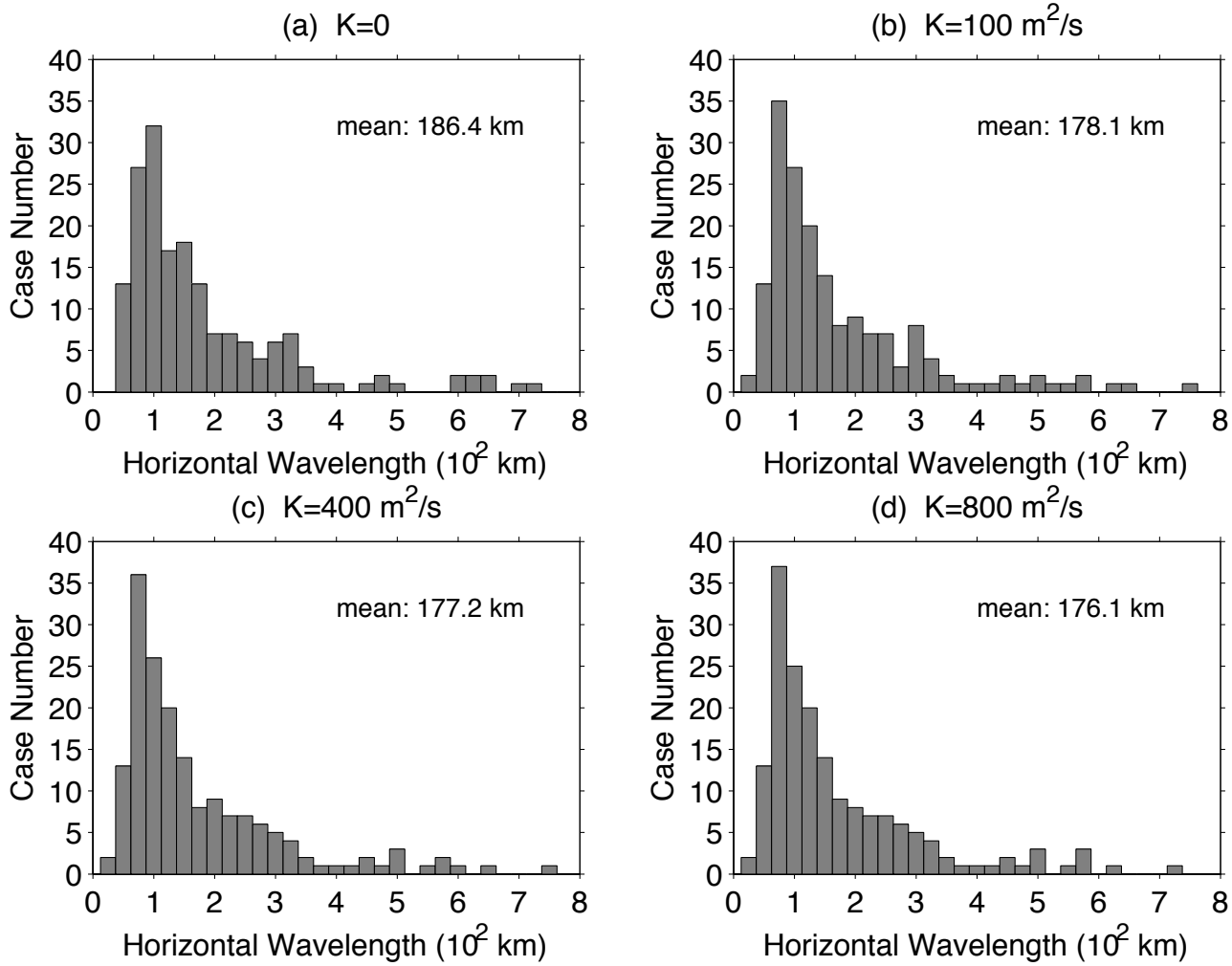
# Intrinsic Periods from Polarization Relation




*Lu et al.*, [2016]

- 77.1%, 78.9%, 80.6%, to 81.1% waves have frequencies shifted to higher-frequency regimes for K=0, 100, 400, 800 m$^2$/s.

# Horizontal Wavelengths

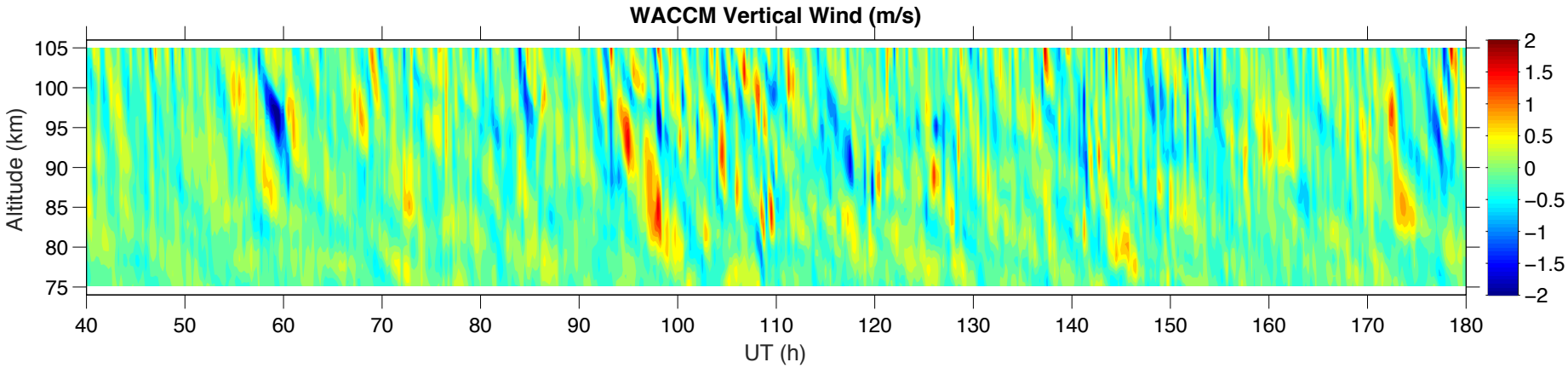



*Lu et al.*, [2016]

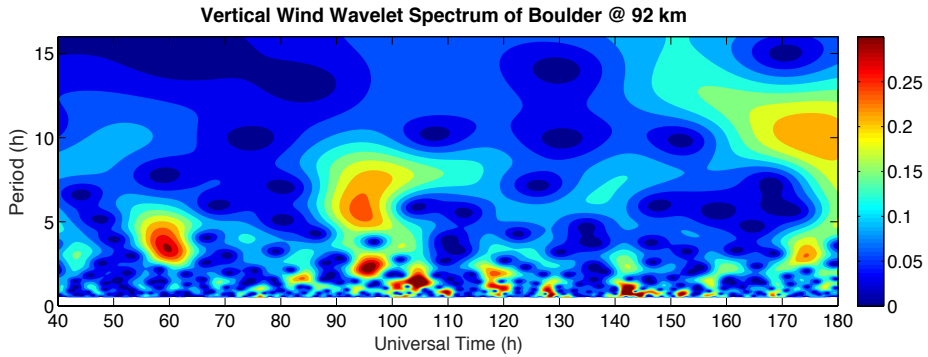
- Mean horizontal wavelengths are about ~180 km.
- Estimated values decrease slightly from for K=0, to K=800 m$^2$/s.

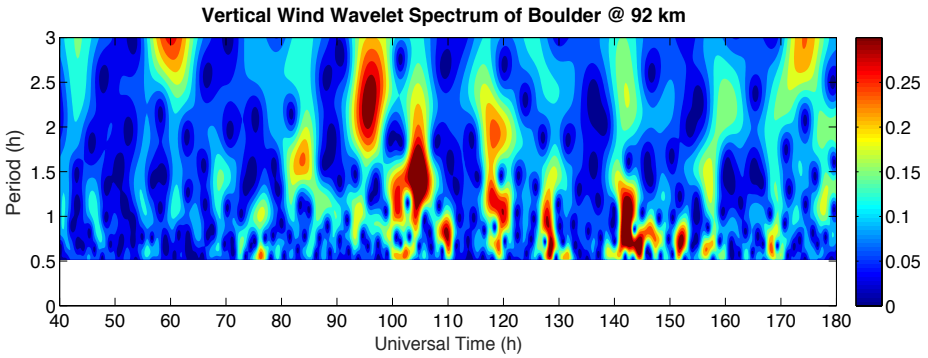
# High-Resolution WACCM

**Raw Vertical Wind:** Δt=15 min, Δz=0.1H, Δx=0.25°




**Wavelet Spectra:**

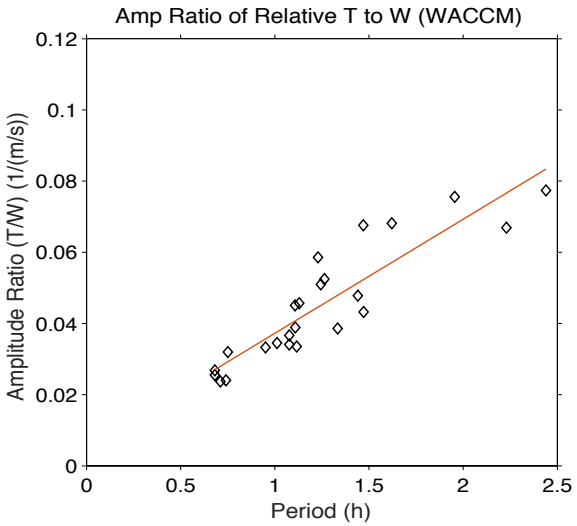


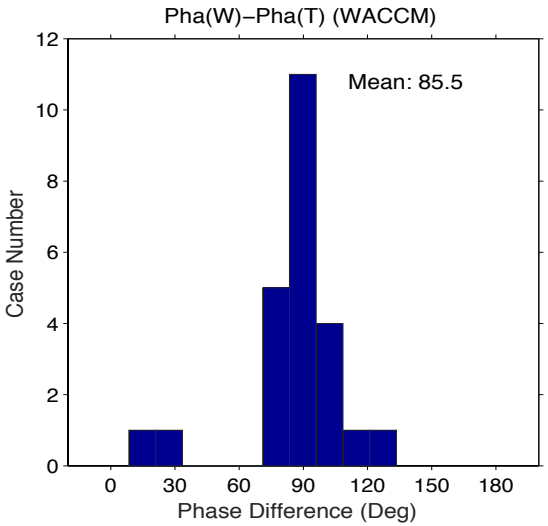



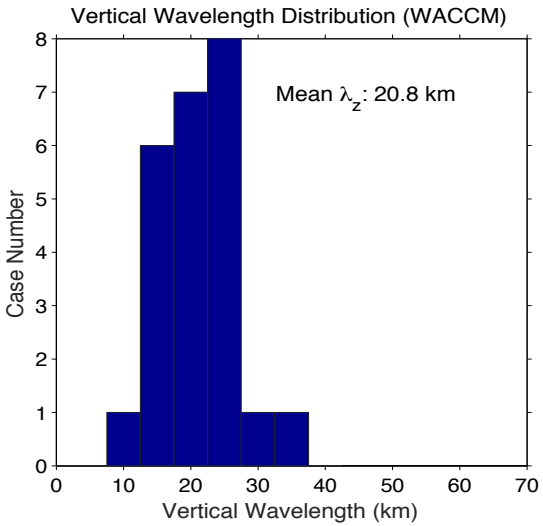

High-to-medium frequency waves are also the prominent features in the high-res WACCM

# WACCM

24 Waves

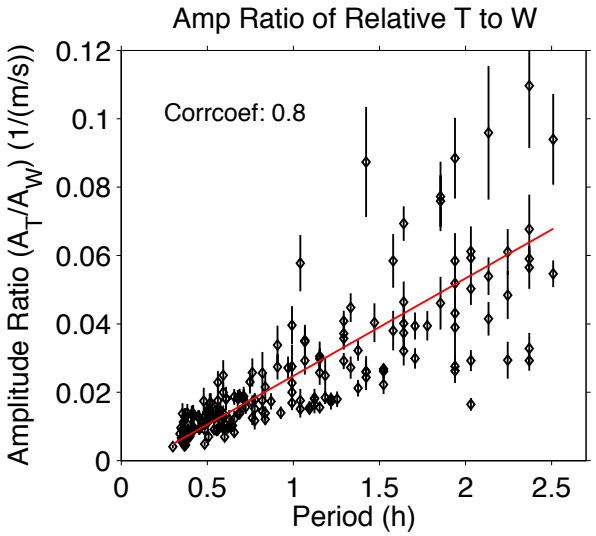


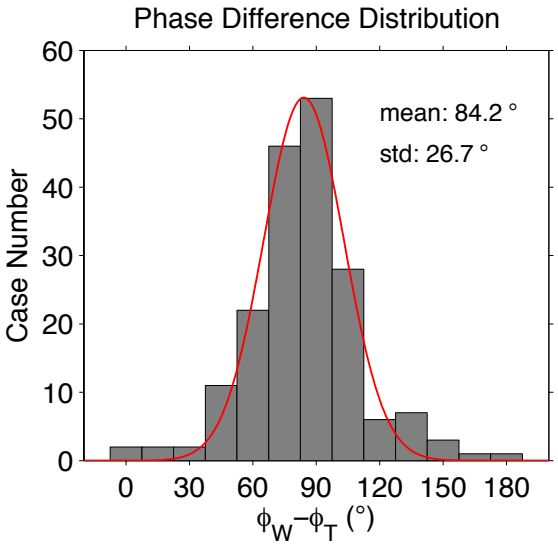



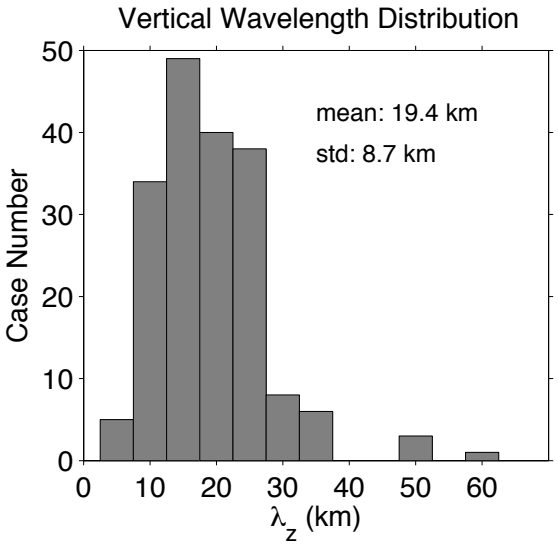




# Lidar

184 Waves



# Summary

- **Focus: The high-to-medium (0.3-2.5h) mesoscale gravity waves**

- **Statistical Results:**

1) **Amplitude ratios (T/W): positively correlate with periods.**
2) **Phase differences (W-T): Gaussian distribution of 84º± 27º.**

**The deviation of phase difference is much larger than that expected for non-dissipative waves. Larger deviation is for shorter $\lambda_z$.**

3) **Vertical wavelengths: 19.4±8.7 km.**
4) **Horizontal wavelengths: mesoscale with a mean of ~180 km.**

- **Comparison with high-resolution WACCM**

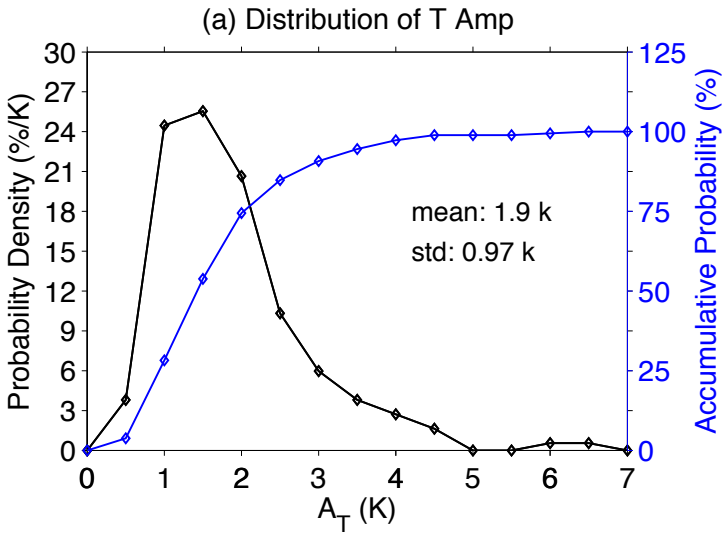
1) **Good agreement is found compared with observations.**
2) **More cases are needed from the model**
3) **Wave sources and propagation are interesting for further study.**

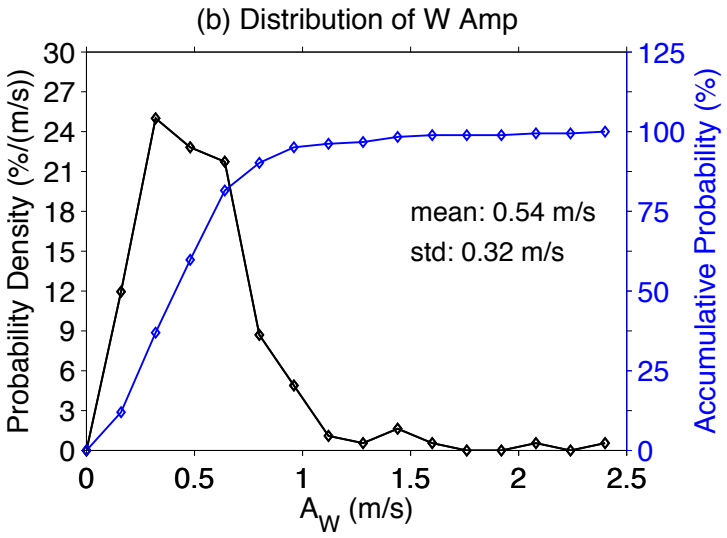

Vadas, S. L., and D. C. Fritts (2005), Thermospheric responses to gravity waves: Influences of increasing viscosity and thermal diffusivity, *J. Geophys. Res.*, *110*, D15103, doi:10.1029/2004JD005574.

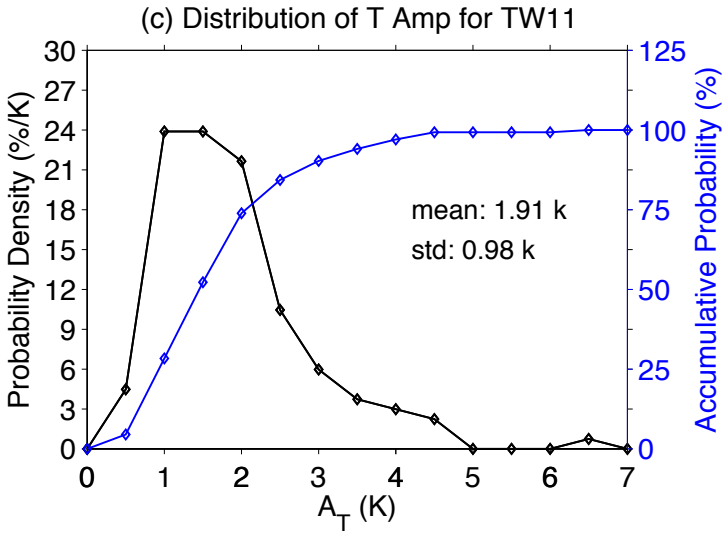
Vadas, S. L., and M. J. Nicolls (2012), The phases and amplitudes of gravity waves propagating and dissipating in the thermosphere: Theory, *JGR.*, *117*, A05322, doi:10.1029/2011JA017426.

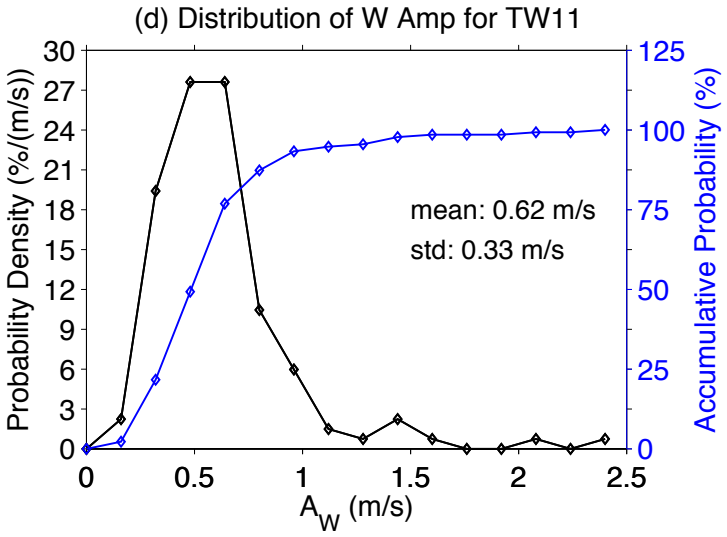
Lu et al., (2016), Statistical characterization of high-to-medium frequency gravity waves by lidar-measured vertical winds and temperatures in the MLT, *JASTP*, LPMT special issue, under review

# Amplitude Distribution



# Data Source

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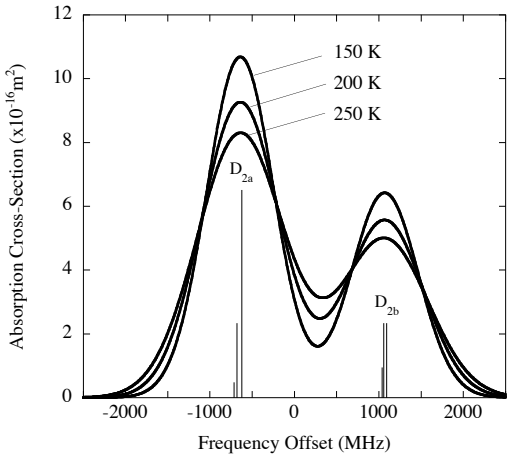
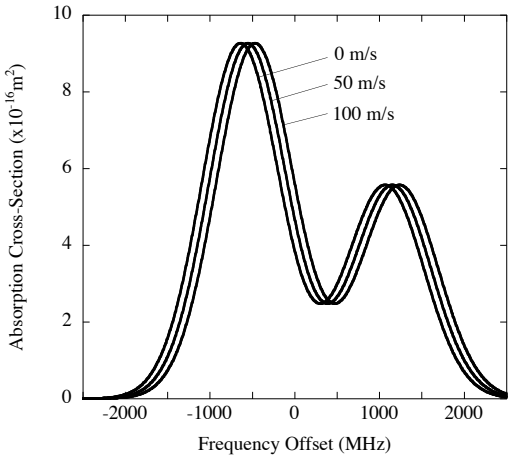
- Data Resolution: Δt=7.5 min, Δz=0.96 km

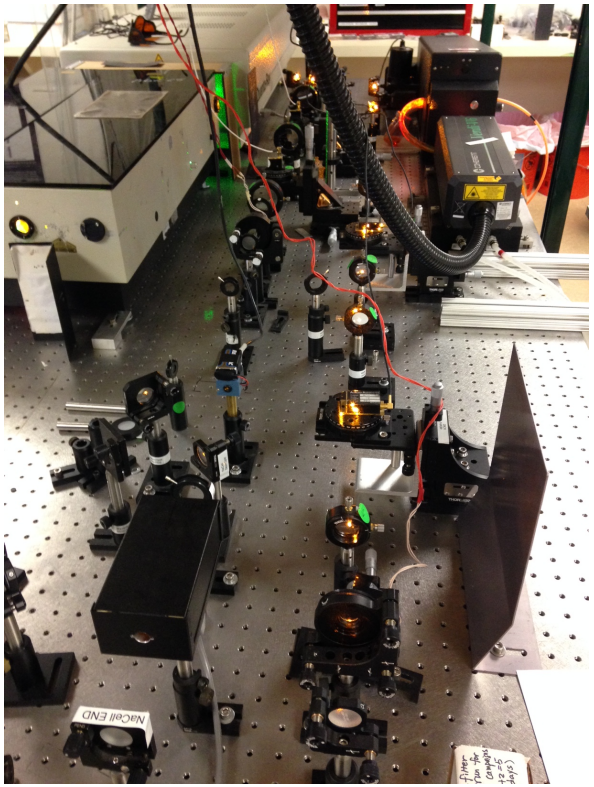
## Na Doppler Lidar Principle

T




W



# Derive Intrinsic frequency without dissipation

$$\omega_{Ir} = \left| \frac{N^2 \left( im - \frac{1}{2H} \right) \Big/ g \left( m - \frac{i}{2H} + \frac{i}{\gamma H} \right)}{\left| \tilde{T} / \overline{T} / \tilde{w} \right|} \right|$$

*Lu et al.*, [2016]

# Derive Intrinsic frequency with dissipation

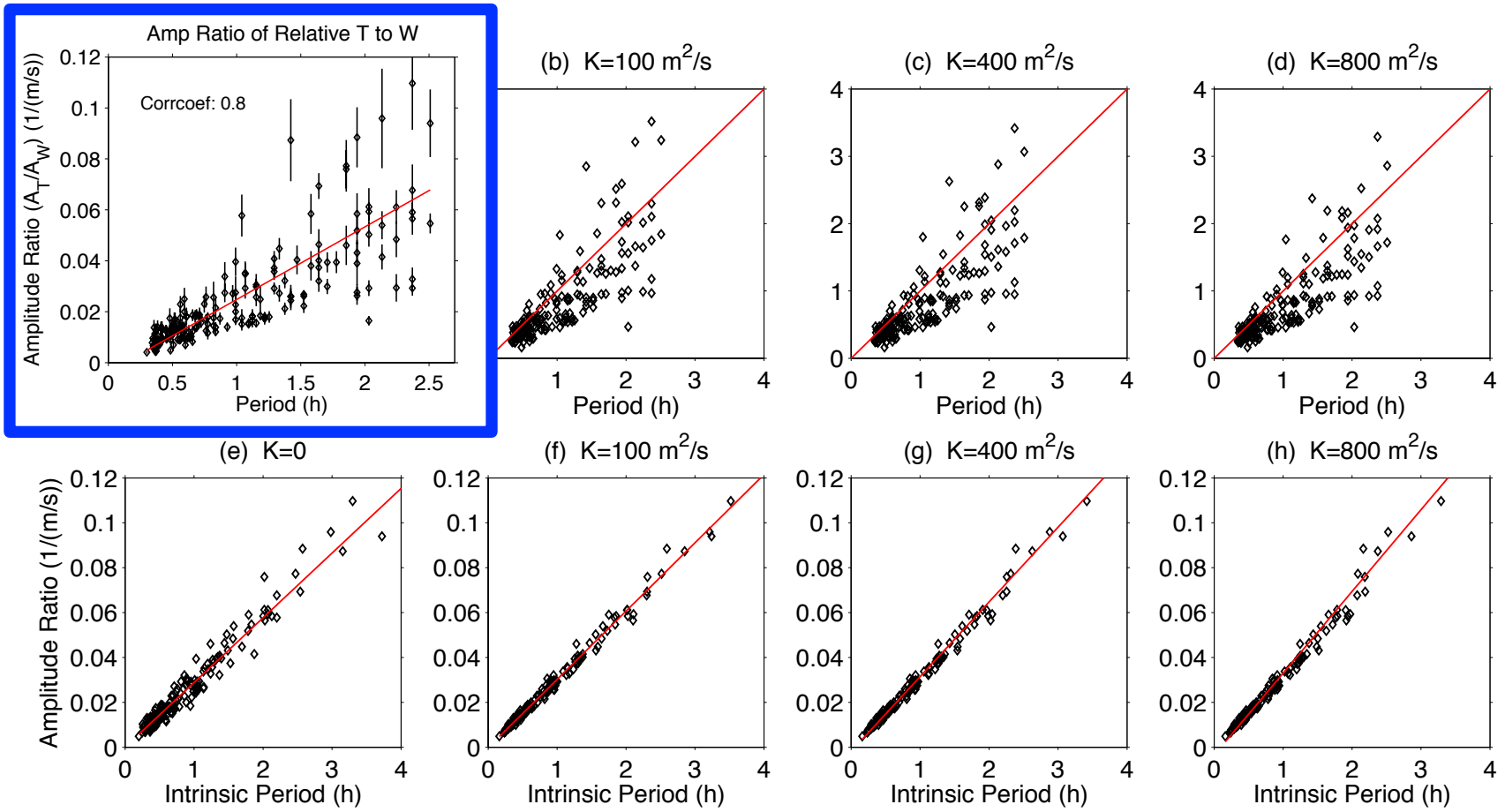
$$\omega_{Ii} = -\frac{K}{2} \left( k_H^2 + m^2 - \frac{1}{4H^2} \right) \frac{\left[ 1 + (1 + 2\delta) / \mathrm{Pr} \right]}{(1 + \delta_+ / 2)} \tag{7}$$

and the dispersion relation involving $\omega_{Ir}$, $k_H$, $m$ and $K$ can be written as (Eq. (26) in VF05),

$$\omega_{Ir}^2 + \frac{K^2}{4} \left( k_H^2 + m^2 - \frac{1}{4H^2} \right) \left( 1 - \mathrm{Pr}^{-1} \right)^2 \frac{(1 + \delta_+ + \delta^2 / \mathrm{Pr})}{(1 + \delta_+ / 2)^2} + \frac{K_+ m \omega_{Ir}}{H} + \frac{K^2 m^2}{\mathrm{Pr}\, H^2} = \frac{k_H^2 N^2}{k_H^2 + m^2 + 1/4H^2} \tag{8}$$

where $\delta_+ = \delta(1 + \mathrm{Pr}^{-1})$, $K_+ = K(1 + \mathrm{Pr}^{-1})$ and $\delta = Km / H\omega_{Ir}$.

# Intrinsic Periods from Polarization Relation




- 77.1%, 78.9%, 80.6%, to 81.1% waves have frequencies shifted to higher-frequency regimes for K=0, 100, 400, 800 m$^2$/s
- As expected, amplitude ratios have a linear relation with intrinsic frequencies.